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Estimation of Aerodynamic Center and Span Load Distributions of Swept Wings

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Nomenclature

a or a_ν = local sectional lift curve slope
AR = aspect ratio
b = wing span
$b_{\nu n}$ = coefficient defined by Eq. (17) of Ref. (9)
c or c_ν = local wing chord, bar denotes average chord
M = Mach number
m = total number of spanwise control stations
n = exponent defined by Eq. (6)
n_0 = exponent defined by Eq. (5)
R = Reynolds number based on mean aerodynamic chord
x_{ac} = aerodynamic center position measured from wing leading edge
y_c or y_t = spanwise coordinate measured relative to wing center or tip, respectively, in terms of local chord
α or α_ν = local geometric angle of attack
γ = nondimensional circulation
η = nondimensional spanwise coordinate
λ = taper ratio
$\lambda(y)$ = parameter defined by Eqs. (1) and (2)
ν = index denoting spanwise location
$\varphi_{c/2}$ = semichord sweep angle
φ_e = effective semichord sweep angle
ω = downwash factor of wing

Introduction

THERE exist several methods for calculating the aerodynamic center distributions on swept wings. It is the purpose of this Note to indicate the most appropriate method, or methods, applicable to swept, moderate aspect ratio wings. Reference 1 presents a method which is given in two forms for determining the aerodynamic center distribution with respect to the quarterchord line of a wing. One form is the tangent approximation and the other form is the hyperbolic approximation. Reference 2 presents a method, based on the Multhopp lifting surface theory of Ref. 3, which includes a correction for wing thickness.

Discussion

The hyperbolic and tangent approximations are given in Ref. 1, as respectively.

$$\lambda(y) = \left[1 + \left(2\pi \frac{\tan\varphi_e}{\varphi_e} y\right)^2\right]^{1/2} - 2\pi \frac{\tan\varphi_e}{\varphi_e} y \qquad (1)$$

and

$$\lambda(y) = 1.40 + 1.33y - (0.16 + 7.30y)^{1/2} \qquad (2)$$

The constants in the Eq. (2) were determined such that the spanwise aerodynamic center position is tangent to the $c/4$-line [i.e., $\lambda(y) = 0$] for y equal to unity.

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Both experimental and theoretical aerodynamic center locations are shown in Figs. 1a, 1b, 1c, and 1d for the wings of Refs. 4, 5, 6, and 7, respectively. These wings are labeled as planforms A, B, C, and D in the subsequent discussion. The geometrical characteristics of the wings are tabulated as follows:

Table 1 Geometrical characteristics of wings

Wing	AR	$\varphi_c/4$, deg	λ	Airfoil section	t/c, %
A	2.828	49.51	0.333	RAE 102	6
B	3.0	45.0	0.5	NACA 64A010	10
C	5.0	45.0	1.0	RAE 101	12
D	8.0	45.0	0.45	NACA 63A012	12

The Multhopp lifting surface theory result obtained for each wing, from a computer program based on Ref. 8, is also included for comparison. The number of control points used in the computer program was 4 chordwise and 31 spanwise (tip to tip). The experimental data shown in Fig. 1 are determined from an average of the experimental values between the angles of attack of 2° and 5°. Examination of Fig. 1 shows that the tangent approximation of Ref. 1 gives a good representation of the aerodynamic center location for all wings except the high aspect ratio wing D. In this case, either the Multhopp lifting surface theory[8] or Transonic Data Memorandum method,[2] without the thickness correction, gives good agreement with experiment. It appears from the experimental results shown in Figs. 1a, 1b, and 1d that the aerodynamic center positions do in fact follow the predicted variation in moving toward the trailing edge at the wing root and toward the leading edge at the tip. Also shown in Fig. 1c is the extended tangent, as recommended in Ref. 2, for representing the aerodynamic center distribution on the outboard portion of the wing.

One question of immediate interest is the significance of the aerodynamic center position on the loading of swept wings using the method suggested by Kuchemann.[1] This method determines the two-dimensional sectional lift curve slope for swept or unswept wings and then calculates the span loading using a quasi-lifting line analysis. The sectional lift slope is given in Ref. 1 as

$$a = a_0 \frac{\cos\varphi_e}{\sin\pi n_0} \frac{2n}{1 - \pi n(\cot\pi n - \cot\pi n_0)} \qquad (3)$$

$$\varphi_e = \varphi_{c/2}/[1 + (a_0 \cos\varphi_{c/2}/\pi A)^2]^{1/4} \qquad (4)$$

$$n_0 = \tfrac{1}{2}[1 - (\varphi_e/\pi/2)] \qquad (5)$$

and

$$n = 1 - \frac{1 + \lambda(y)(\varphi_e/\pi/2)}{2[1 + (a_0 \cos\varphi_e)/\pi A)^2]^{1/4[1+|\varphi_e|/(\pi/2)]}} \qquad (6)$$

The term $\lambda(y)$ is related to the aerodynamic center location by the expression

$$\lambda(y) = 2\pi(x_{ac} - 0.25)/\varphi_e \qquad (7)$$

The integral equation for the spanwise loading is given as

$$\frac{2b}{a(\eta)c(\eta)} \gamma(\eta) = \alpha(\eta) - \frac{\omega}{2\pi} \int_{-1}^{1} \frac{d\gamma(\eta')}{d\eta'} \frac{d\eta'}{\eta - \eta'} \qquad (8)$$

This equation can be solved by the Multhopp integration method[9] which results in the system of linear equations

$$\gamma_\nu \left(b_{\nu\nu} + \frac{2b}{\omega a_\nu c_\nu}\right) = \frac{\alpha_\nu}{\omega} + \sum_{n=1}^{m}{}' b_{\nu n}\gamma_n \qquad (9)$$

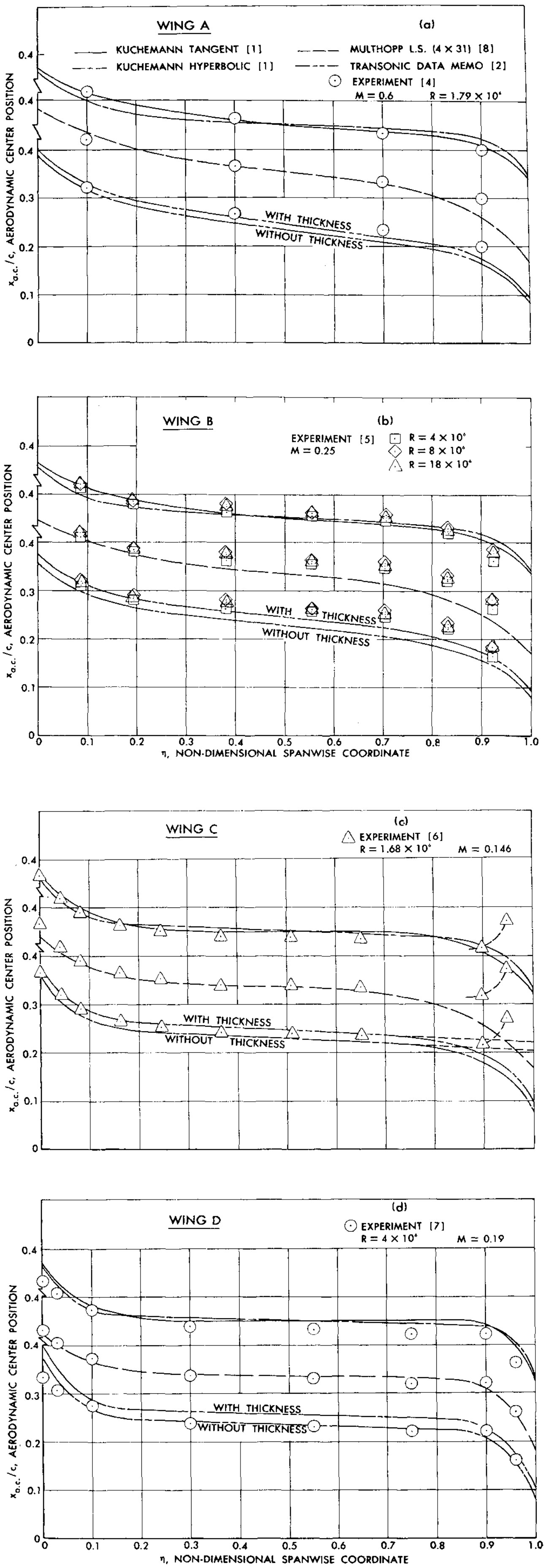

Fig. 1 Aerodynamic center position as a function of the nondimensional spanwise coordinate.

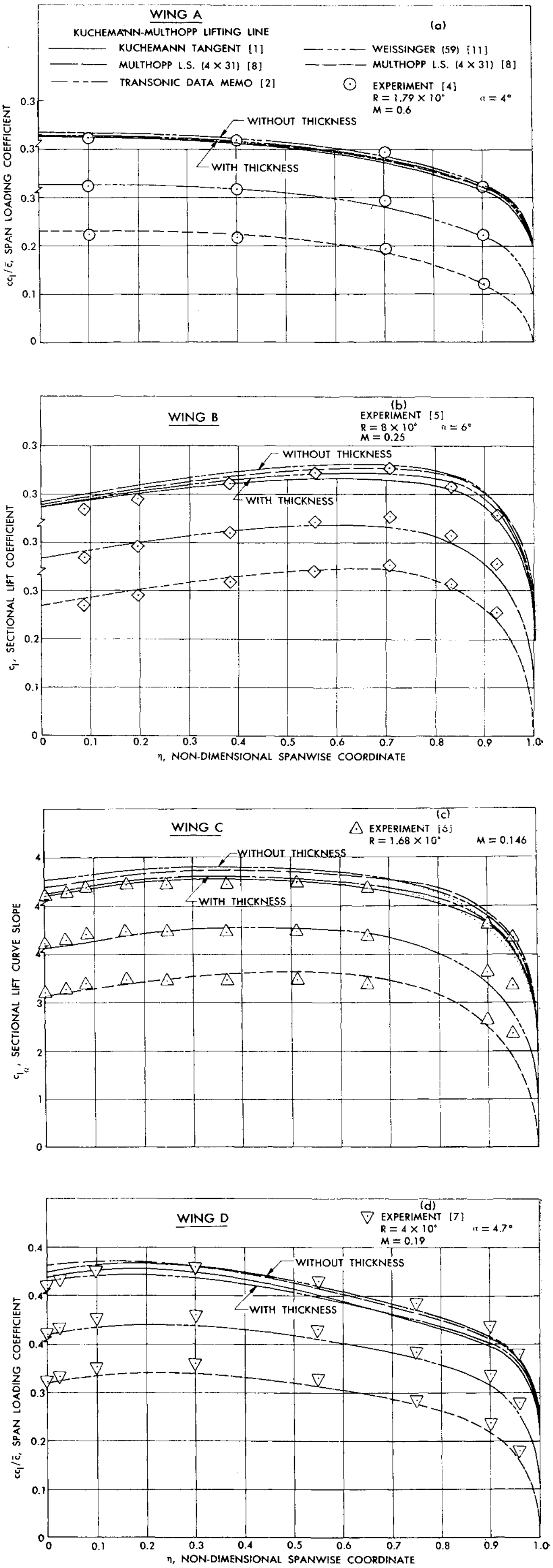

Fig. 2 Span loading characteristics as a function of the nondimensional spanwise coordinate.

The prime symbol denotes exclusion from the sum of the quantities having the same subscripts ν and n. The downwash factor ω is defined by the relation

$$\omega = 2n \tag{10}$$

Reference 1 takes an average value of ω equal to $2n_s$, which is the sheared value for a swept wing $[\lambda(y) = 0]$. For a 45° swept wing with aspect ratios of 3, 5, and 7, the value for ω is 1.025, 1.010, and 1.005, respectively. Strictly speaking, ω is also dependent on the spanwise location and would vary in the same manner as n varies, since $\omega = 2n$.

The load distribution obtained by using Eq. (9), for $m = 31$, is shown in Figs. 2a, 2b, 2c, and 2d for the wing planforms of Refs. 4, 5, 6, and 7, respectively. In accordance with Refs. 1 and 10, the value for ω was held constant; a value of unity was assigned following Ref. 10. Figure 2 shows the Kuchemann-Multhopp lifting line method using aerodynamic centers obtained by the methods of Refs. 1, 2, and 8 on the upper set of curves in each figure. The difference in load distribution between that calculated using the Kuchemann tangent approximation and the Kuchemann hyperbolic approximation is small, so that the loading curves shown make use of the tangent approximation. In addition, no consideration was given to wing thickness effects, except in its effect on the aerodynamic center position in using the method of Ref. 2, in the theoretical calculations so the value of a_0 used was 2π. In general, of the loadings obtained by the Kuchemann-Multhopp lifting line method, that obtained from using the aerodynamic centers from the Kuchemann tangent approximation appears to give the best agreement with experiment except for the case of wing D. The loading for wing D appears to be best represented by use of either the aerodynamic centers from the Multhopp lifting surface theory[8] or the Transonic Data Memorandum method[2] without the thickness correction. The dotted curves shown in Fig. 2c show the slight reduction in the calculated loading obtained by using the extended aerodynamic center distribution shown by the dashed lines in Fig. 1c.

Figure 2 also shows the span loading characteristics of wings A, B, C, and D as obtained from the Weissinger[11] (using 59 spanwise points) and Multhopp lifting surface[8] (using 4 chordwise and 31 spanwise points) theories compared with experiment. This figure shows that the loadings predicted by the Weissinger theory and Multhopp lifting surface theory give comparable results and agree quite well with experiment except in the case of wing D where both theories slightly underpredict the level of the loading on the wing.

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Change in Pitching-Moment Coefficient Due to Ground Effect

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Nomenclature

a_t = slope of tail lift coefficient
C_L = lift coefficient
C_m = pitching-moment coefficient
$\bar{c}$ = mean aerodynamic chord
q = dynamic pressure
S = wing area
α = angle of attack
Δ = denoting the change due to ground effect in case of constant angle of attack, e.g., $\Delta C_L = C_L - C_{L_0}$
δ_e = elevator angle
ϵ = downwash angle at the tail
τ = relative control effectiveness, $\tau = \partial\alpha_t/\partial\delta_e$

Subscripts

t = tail
w = wing-body combination
0 = free air, e.g., C_{L_0}
$|_\delta$ = constant elevator angle

REFERRING to a new flight-test method for measurement of ground effect of fixed wing aircraft,[1] this Note is concerned with the change in pitching-moment coefficient due to ground effect and the factors causing this change in case of a constant-angle-of-attack approach. The investigation described herein has been derived from a method which was used for evaluating the change in downwash angle at the tail when measuring ground effect of the Transall C-160 airplane.[2]

Dividing up the forces and moments as shown in Fig. 1, the change in lift coefficient due to ground effect can be expressed as

$$\Delta C_L = \Delta C_{L_w} + k_t(\Delta C_{L_t}|_\delta + \tau a_t \Delta\delta_e) \tag{1}$$

where terms of small magnitude are neglected and

$$k_t = S_t q_{t0}/S q_0 = S_t(q_{t0} + \Delta q_t)/[S(q_0 + \Delta q)] \tag{2}$$

is assumed to be constant. The difference between the moment equations in free air and in ground proximity yields

$$x_a\,\Delta C_{L_w} - k_t(l'_t - x_a)(\Delta C_{L_t}|_\delta + \tau a_t \Delta\delta_e) = 0 \tag{3}$$

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